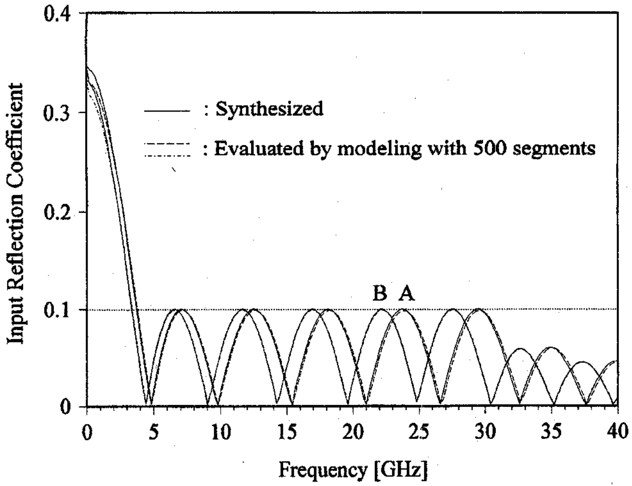

Fig. 4. Synthesized frequency characteristics in the lossy case.

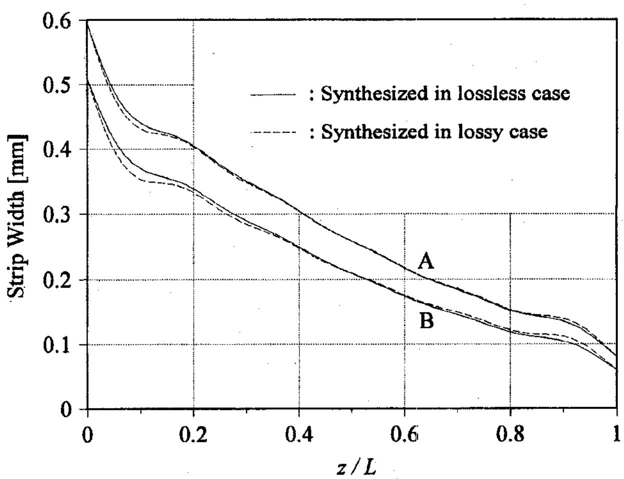

Fig. 5. Synthesized strip width in the lossy case ($f = 0$).

long as ρ_i is small. The exact numerical results in this paper show that the present theory provides a generalized theory for determining the impedance taper profile in lossy and dispersive media.

IV. Conclusion

A new efficient synthesis technique for the specified frequency response of lossy and dispersive tapered transmission line has been presented. This technique was accomplished by the optimization process to extract the optimum null points for the synthesis of the desired taper profile in the existence of a loss and dispersion. The results of synthesizing a microstrip transformer for example shows that the present synthesis technique with loss is important for design of high-frequency and high-density integrated circuits giving effect on the determination of the electrical length.

References

[1] R. W. Klopfenstein, "A transmission line taper of improved design," *Proc. IRE*, vol. 44, pp. 31–35, Jan. 1956.

[2] P. Pramanick and P. Bhartia, "A generalized theory of tapered transmission line matching transformers and asymmetric couplers supporting non-TEM modes," *IEEE Trans. Microwave Theory Tech.*, vol. 37, pp. 1184–1191, Aug. 1989.

[3] J. P. Mahon and R. S. Elliott, "Tapered transmission lines with a controlled ripple response," *IEEE Trans. Microwave Theory Tech.*, vol. 38, pp. 415–1420, Oct. 1990.

[4] M. Kobayashi and N. Sawada, "Analysis and synthesis of tapered microstrip transmission lines," *IEEE Trans. Microwave Theory Tech.*, vol. 40, pp. 642–1646, Aug. 1992.

[5] R. E. Collin, *Foundations for Microwave Engineering*. New York: McGraw-Hill, 1966.

[6] R. F. Hyneman, "A technique for the synthesis of line-source antenna patterns having specified sidelobe behavior," *IEEE Trans. Antennas Propagat.*, vol. AP-16, pp. 430–435, July 1968.

[7] R. A. Pucel, D. J. Mass, and C. P. Hartwig, "Losses in microstrip," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-16, pp. 342–350, June 1968.

[8] I. J. Bahl and D. K. Trivedi, "A designer's guide to microstrip," *Microwaves*, vol. 16, no. 5, pp. 174–182, May 1977.

[9] M. Kirchning and R. H. Jansen, "Accurate model for effective dielectric constant of microstrip with validity up to millimeter-wave frequencies," *Electron. Lett.*, vol. 18, pp. 272–273, 1982.

Mode Orthogonality Relations and Field Structure in Chirowaveguides

E. O. Kamenetskii

***Abstract*—By analyzing the vector and scalar equations for chirowaveguides, two forms of mode orthogonality relations are obtained: the vector formulated orthogonality and the scalar formulated orthogonality. The first one is applicable to the general case of open chiroplasma or chiroferrite waveguides. It is shown that for two parallel-plate isotropic chirowaveguides, these two forms of orthogonality relations differ. Based on mode orthogonality relations, it is shown that in chirowaveguides the polarization of so-called complex modes differs from that of propagating or evanescent modes. The correlation between field components of two complex modes that transfer active power flow in chirowaveguides is obtained.**

I. Introduction

A number of problems related to chirowaveguides have been investigated and reported [1]–[13]. For example, dispersion characteristics and field distributions in parallel-plate [1]–[3], open-slab [4], [5], circular [2], [6], [7], and closed rectangular [8], [9] chirowaveguides have been studied. The surface waves in chiral layers have been analyzed noting elliptically polarized transverse electric and magnetic fields in the layers [10]. The theory of wave propagation in chiroplasma and chiroferrites [11] and the theory of chiroferrite waveguides [12], have also appeared. It has been pointed out in [13] that modes in chirowaveguides have interesting and useful properties of power orthogonality.

The power orthogonality (or vector formulated orthogonality relations) obtained in [13] for isotropic chirowaveguides may be easily extended to a more general case of lossless open chiroplasma or chiroferrite waveguides. Together with this type of orthogonality, one can also obtain the scalar formulated orthogonality relations. We will show that the scalar formulated orthogonality are not derived from the vector formulated orthogonality.

Manuscript received January 4, 1995; revised November 27, 1995.
The author is with the Faculty of Engineering, Tel-Aviv University, Israel.
Publisher Item Identifier S 0018-9480(96)01552-9.

0018–9480/96$05.00 © 1996 IEEE

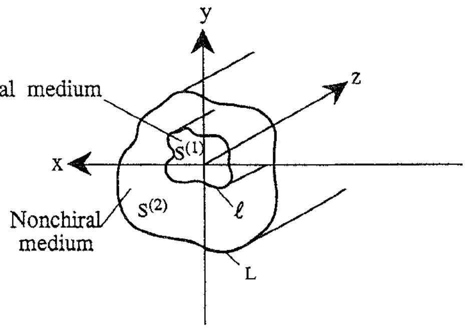

Fig 1. A cylindrical waveguide.

Mode orthogonality relations obtained in this paper are correct for different types of modes: propagating, evanescent, or complex. Complex modes have been found recently in closed chirowaveguides [3], [7], [9]. We will show that the polarization of complex modes in chirowaveguides differs from the polarization of propagating and evanescent modes. An interesting result obtained in this paper is a special type of reflection symmetry for complex modes. This reflection symmetry shows the correlation between field components for two complex modes that transfer active power flow in chirowaveguides.

The relations of mode orthogonality are useful for solving mode excitation problems and for the development of coupled-mode formalism in chirowaveguides. Such relations allow us to obtain correlation between mode field components and therefore to give information about a structure of mode fields without solving the dispersion equations.

II. Vector and Scalar Formulated Orthogonality Relations

One can use an abstract formulation to write the time-harmonic fields in a regular waveguide

$$G\vec{\Psi} = 0 \tag{1}$$

where G is a linear differential operator, and $\vec{\Psi}$ is a vector wave function that characterizes the fields. The function $\vec{\Psi}$ satisfies homogeneous boundary conditions. To obtain the orthogonality relation, we have to define the scalar product for two vector functions and to introduce a notion of conjugate operator [14]. In the vector-formulated orthogonality relations, the vector wave function $\vec{\Psi}$ is composed by the vectors of the electromagnetic field. In the scalar-formulated orthogonality relations, one has some scalar components of the fields as components of the function $\vec{\Psi}$. In the case of chirowaveguides, it will be shown that the scalar formulated orthogonality relations are not derived from the vector formulated relations. These two types of orthogonality complement each other.

A. *Vector-Formulated Orthogonality*

The vector formulated orthogonality relations in chirowaveguides are directly derived from Maxwell equations. Fig. 1 shows a cylindrical waveguide with an arbitrary cross-sectional shape that is filled with chiral material characterized by the chirality admittance ν (the cross-section $S^{(1)}$) and nonchiral material (the cross-section $S^{(2)}$). The contour ℓ separates chiral and nonchiral media. The external contour L surrounding the waveguide cross-section $S[S = S^{(1)} + S^{(2)}]$ is an electric or magnetic wall. In the case of an open waveguide the boundary L is at infinity.

For every region $q(q = 1, 2)$ of the waveguide we write Maxwell equations for time-harmonic fields $(e^{i\omega t})$ [1]

$$M^{(q)}\mathbf{U}^{(q)} = 0 \tag{2}$$

where $M^{(q)}$ are Maxwell operators

$$M^{(q)} = \begin{pmatrix} i\omega\{\epsilon^{(q)} + \mu[\nu^{(q)}]^2\} & -[(\nabla\times) - \omega\mu\nu^{(q)}] \\ (\nabla\times) - \omega\mu\nu^{(q)} & i\omega\mu \end{pmatrix} \tag{3}$$

(here we have $\nu^{(1)} = \nu$, $\nu^{(2)} = 0$). $\mathbf{U}^{(q)}$ is a vector function of fields

$$\mathbf{U}^{(q)} = \begin{pmatrix} \mathbf{E}^{(q)} \\ \mathbf{H}^{(q)} \end{pmatrix}. \tag{4}$$

Let γ_m and $\mathbf{U}_m^{(q)}$ be, correspondingly, the propagation constant and the fields of mode m of the main boundary problem and $\tilde{\gamma}_n$ and $\tilde{\mathbf{U}}_n^{(q)}$ are correspondingly the propagating constant and the fields of mode n of the conjugate boundary problem. One can obtain the following relation [13], [15]

$$(\gamma_m + \tilde{\gamma}_n^*) \sum_{q=1}^{2} \int_{S^{(q)}} [Q\hat{\mathbf{U}}_m^{(q)}] \cdot [\tilde{\hat{\mathbf{U}}}_n^{(q)}]^* \, ds = 0. \tag{5}$$

In this relation, two modes are orthogonal for $\gamma_m + \tilde{\gamma}_{\tilde{n}}^* \neq 0$. If, however, $\gamma_m + \tilde{\gamma}_{\tilde{n}}^* = 0$, we will mark $n = \tilde{m}$ and consider this mode as the conjugate mode to the mode m. For conjugate modes we have an expression for the norm

$$\begin{aligned} N_m &\equiv N_{m\tilde{m}} \\ &= \sum_{q=1}^{2} \int_{S^{(q)}} [Q\hat{\mathbf{U}}_m^{(q)}] \cdot [\hat{\mathbf{U}}_{\tilde{m}}^{(q)}]^* \, ds \\ &= \sum_{q=1}^{2} \int_{S^{(q)}} [\hat{\mathbf{E}}_m^{(q)} \times \mathbf{H}_{\tilde{m}}^{(q)*} + \hat{\mathbf{E}}_{\tilde{m}}^{(q)*} \times \hat{\mathbf{H}}_m^{(q)}] \cdot \mathbf{e}_z \, ds \end{aligned} \tag{6}$$

which describes the active power flow in a waveguide. The expressions (5) and (6) are correct for propagating modes (γ is an imaginary quantity) and for evanescent (γ is a real quantity) and complex modes in closed lossless waveguides.

Complex modes in chirowaveguides have been investigated only for lossless reciprocal waveguide structures [3], [7], [9]. There are no publications at present concerning complex modes in lossless chiroplasma or chiroferrite waveguides. The spectrum of complex modes in reciprocal chirowaveguides has symmetrical positions on the complex plane similar to the spectrum of complex modes in shielded lossless dielectric waveguides (see Fig. 2). An interaction of modes with γ_m and $-\gamma_m^*$ (or $-\gamma_m$ and γ_m^*) causes active power flow, meanwhile an interaction of modes with γ_m and γ_m^* (or $-\gamma_m$ and $-\gamma_m^*$) leads to reactive power flow [16]–[18]. The pairs of modes that realize the carrying over of energy are characterized by the same direction of phase velocity and different sign of amplitude changing. Transmission of energy by complex modes is possible only at a certain distance, analogous to bellow-cutoff waveguides [17]. The pairs of conjugated modes are symmetrical with respect to the axis β (see Fig. 2).

We can extend the derivation of the vector-formulated orthogonality relations for a general case of open chiroplasma or chiroferrite waveguides. On the basis of the procedure used in [13], [15], one can be convinced that for Hermitian tensors, $\epsilon^{(q)}$ and μ and real quantity ν, we have the same relations (5) and (6).

We apply now the relations of the vector-formulated orthogonality for the special case of two parallel-plate waveguides partially loaded with an isotropic chiral slab (Fig. 3). The thickness of the chiral slab is a and that of the adjacent nonchiral medium is b. The plates separation is $d = a + b$. The fields are independent of the X-axis. We have from (5) and (6) for unit-width waveguide

$$\begin{aligned} &\sum_{q=1}^{2} \int \left[(\tilde{\gamma}_n^* - \gamma_m) E_{m_x}^{(q)} \tilde{E}_{n_x}^{(q)*} - k_d^2 \left(\frac{1}{\tilde{\gamma}_n^*} - \frac{1}{\gamma_m} \right) E_{m_y}^{(q)} \tilde{E}_{n_y}^{(q)*} \right] dy \\ &= \delta_{mn} N_m \end{aligned} \tag{7}$$

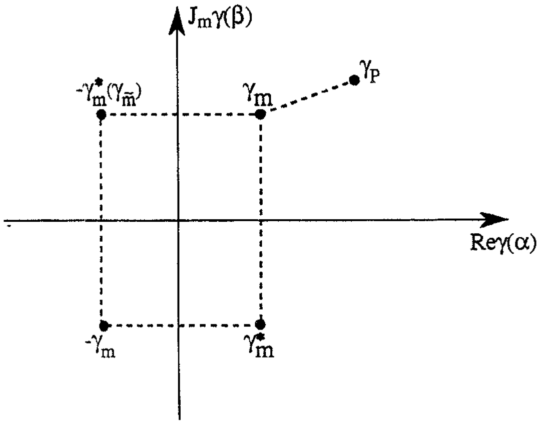

Fig. 2. Positions of complex wave numbers on a complex plane for shielded lossless isotropic waveguide.

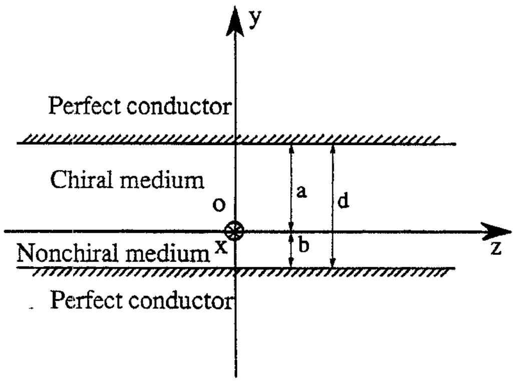

Fig. 3. A two parallel-plate isotropic waveguide.

where $\delta_{mn} = 1$ if $\gamma_m + \tilde{\gamma}_n^* = 0$ and $\delta_{mn} = 0$ if $\gamma_m + \tilde{\gamma}_n^* \neq 0$, $k_d = \omega\sqrt{\epsilon\mu}$. The integration is over the waveguide cross section (over the height y from $-b$ to a). Here we supposed, for simplicity, that $\epsilon^{(1)} = \epsilon^{(2)} = \epsilon$ and used the relations for the fields in chiral medium

$$H_x^{(1)} = i\nu E_x^{(1)} - i\,\frac{\omega\epsilon}{\gamma}\,E_y^{(1)},$$
$$H_y^{(1)} = -i\,\frac{\gamma}{\omega\mu}\,E_x^{(1)} - i\nu E_y^{(1)}. \qquad (8)$$

Analogous relations (for $\nu = 0$) we have for nonchiral medium.

One obtains the following expression for the norm

$$N_m = -\frac{i2}{\omega\mu}\sum_{q=1}^{2}\int\left[\gamma_m E_{m_x}^{(q)} E_{\tilde{m}_x}^{(q)^*} - \frac{k_d^2}{\gamma_m} E_{m_y}^{(q)} E_{\tilde{m}_y}^{(q)^*}\right] dy. \qquad (9)$$

Together with the relation (7), one can also obtain another relation from (5). Using (8) and the analogous relation for nonchiral medium, we have for the structure shown on Fig. 3

$$[\gamma_m^2 - (\tilde{\gamma}_n^*)^2]\sum_{q=1}^{2}\int\left(E_{m_x}^{(q)}\tilde{E}_{n_x}^{(q)^*} + \frac{k_d^2}{\gamma_m\tilde{\gamma}_n^*} E_{m_y}^{(q)}\tilde{E}_{n_y}^{(q)^*}\right) dy = 0. \qquad (10)$$

For $\gamma_m \pm \tilde{\gamma}_n^* = 0$ we obtain

$$N'_m = \sum_{q=1}^{2}\int\left(E_{m_x}^{(q)} E_{\tilde{m}_x}^{(q)^*} \mp \frac{k_d^2}{\gamma_m^2} E_{m_y}^{(q)} E_{\tilde{m}_y}^{(q)^*}\right) dy. \qquad (11)$$

Contrary to the norm defined by (9), the norm N'_m does not have any physical meaning. If $|\gamma_m| \neq |\tilde{\gamma}_n|$ we can write

$$\sum_{q=1}^{2}\int\left(E_{m_x}^{(q)}\tilde{E}_{n_x}^{(q)^*} + \frac{k_d^2}{\gamma_m\tilde{\gamma}_n^*} E_{m_y}^{(q)}\tilde{E}_{n_y}^{(q)^*}\right) dy = 0. \qquad (12)$$

B. Scalar-Formulated Orthogonality

We will obtain the scalar-formulated orthogonality relations for a two parallel-plate waveguide without a nonchiral layer ($b = 0$ on Fig. 3). For an isotropic chirowaveguide, we can write the next vector equation

$$\nabla \times \nabla \times \mathbf{F} - 2\omega\mu\nu \nabla \times \mathbf{F} - k_d^2\mathbf{F} = 0 \qquad (13)$$

which is derived from the chiral constitute relations and Maxwell equations [1]. Here $\mathbf{F}$ is a vector $\mathbf{E}$ or $\mathbf{H}$. From the constitutive relations and Maxwell equation, it also follows that [6], [9]

$$\nabla \cdot \mathbf{F} = 0. \qquad (14)$$

Equations (13) and (14) are correct for every mode that satisfy the vector-formulated eigenvalue problem. After some transformations we have for mode m

$$\Phi_m V_m = 0 \qquad (15)$$

where

$$\Phi_m = \frac{d^2}{dy^2} I + C_m \qquad (16)$$

is the differential-matrix operator

$$C_m = \begin{pmatrix} -(\gamma_m^2 + k_d^2 + 4\omega^2\mu^2\nu^2) & 2\omega\mu\nu\dfrac{k_d^2}{\gamma_m} \\ 2\omega\mu\nu\gamma_m & -(\gamma_m^2 + k_d^2) \end{pmatrix}, \qquad (17)$$

$$V_m = \begin{pmatrix} F_{m_x} \\ F_{m_y} \end{pmatrix} \qquad (18)$$

is the vector function of the fields, I is the unit matrix.

Let us also have mode p with propagation constant γ_p (Fig. 2). Analogously to homogeneous equations (15) we can obtain for mode p

$$\Phi_p V_p = 0 \qquad (19)$$

where the differential-matrix operator Φ_p and the vector function V_p have the same form as the operator Φ_m and the function V_m (we have to put p instead of m).

We define the scalar product as

$$\int_S V_m V_p^*\, ds = \int_S (F_{m_x}F_{p_x}^* + F_{m_y}F_{p_y}^*)\, ds \qquad (20)$$

where S is a cross section area of a waveguide. Now we scalarly multiply (15) by the V_p^* from the right and the complex-conjugate form of (19) by the V_m from the left. After subtracting the last result from the first one, we have

$$\int_o^a \left[\left(\frac{d^2V_m}{dy^2}\right)V_p^* - V_m\left(\frac{d^2V_p}{dy^2}\right)^*\right] dy + \int_o^a [(C_mV_m)V_p^* - V_m(C_pV_p)^*] dy = 0 \qquad (21)$$

where the matrix C_p is similar to the matrix C_m [see (17)]. Using an integration by parts we obtain

$$\int_o^a \left[\left(\frac{d^2V_m}{dy^2}\right)V_p^* - V_m\left(\frac{d^2V_p}{dy^2}\right)^*\right] dy = \left(F_{p_x}^*\frac{dF_{m_x}}{dy} + F_{p_y}^*\frac{dF_{m_y}}{dy} - F_{m_x}\frac{dF_{p_x}^*}{dy} - F_{m_y}\frac{dF_{p_y}^*}{dy}\right)\Bigg|_o^a. \qquad (22)$$

Because of boundary conditions for the electric field on perfect electric walls at $y = 0, a[E_{m_x} = E_{p_x} = 0, (dE_{m_y}/dy) = (dE_{p_y}/dy) = 0]$, the right-hand side of (22) is equal to zero. (We have analogous results for the magnetic field on perfect magnetic walls.) On the basis of (21), one obtains the following relation of orthogonality

$$\int_o^a \left\{[\gamma_m^2 - (\gamma_p^*)^2](E_{m_x}E_{p_x}^* + E_{m_y}E_{p_y}^*) + 2\omega\mu\nu \cdot \frac{k_d^2 - \gamma_m\gamma_p^*}{\gamma_m\gamma_p^*}(\gamma_m E_{m_x}E_{p_y}^* - \gamma_p^* E_{m_y}E_{p_x}^*)\right\} dy = 0. \quad (23)$$

It is evident, that the vector formulated orthogonality relations obtained for parallel-plate chirowaveguide are different from the relation (23).

III. Field Structure

The orthogonality relations enable us to obtain some correlations between mode field components. In particular, such correlations are interesting for complex modes. For previously considered two parallel-plate isotropic chirowaveguides (Fig. 3), we have only the norm for $\gamma_m = -\gamma_n^*$ [see (9)]. Evidently, for $\gamma_m = \gamma_n^*$, the norm defined from (7) will be equal to zero. This is a special feature of complex modes in chirowaveguides in comparison with shielded dielectric waveguides [16]–[18].

To obtain the norm (9) as a real quantity, and thus to obtain active power flow provided by two complex modes, we have to demand that

$$E_{\tilde{m}_x}^{(q)} = \frac{-i\gamma_m}{|\gamma_m|} E_{m_x}^{(q)},$$
$$E_{\tilde{m}_y}^{(q)} = \frac{i|\gamma_m|}{\gamma_m} E_{m_y}^{(q)}. \quad (24)$$

If we substitute (24) into (9) we have the real norm N_m

$$N_m = \frac{2}{\omega\mu} \sum_{q=1}^{2} \int \left(|\gamma_m| E_{m_x}^{(q)} E_{m_x}^{(q)*} + \frac{kd^2}{|\gamma_m|} E_{m_y}^{(q)} E_{m_y}^{(q)*}\right) dy. \quad (25)$$

For $\gamma_m = \alpha_m + i\beta_m$ we obtain from (24)

$$\frac{E_{\tilde{m}_x}^{(q)}}{E_{\tilde{m}_y}^{(q)}} = -\left(\frac{\alpha_m^2 - \beta_m^2}{\alpha_m^2 + \beta_m^2} + i\frac{2\alpha_m\beta_m}{\alpha_m^2 + \beta_m^2}\right)\frac{E_{m_x}^{(q)}}{E_{m_y}^{(q)}}. \quad (26)$$

We can see that for any kinds of polarization of complex mode m, mode $\tilde{m}$ is elliptically polarized and may be described as

$$\frac{E_{\tilde{m}_x}^{(q)}}{E_{\tilde{m}_y}^{(q)}} = c + id \quad (27)$$

where c and d are real coefficients.

Because of similarity of modes m and $\tilde{m}$, we can assert that mode m is generally described as

$$\frac{E_{m_x}^{(q)}}{E_{m_y}^{(q)}} = a + ib \quad (28)$$

where a and b are real coefficients. Therefore, the complex modes in parallel plate isotropic chirowaveguides that provide active power flow have the elliptical polarization of the type (28).

It is necessary to note that according to [1] for propagating (γ_m is an imaginary quantity) modes in chiral slab-waveguides we have the following type of elliptical polarization

$$\frac{E_{m_x}}{E_{m_y}} = iA \quad (29)$$

where A is a real quantity.

On the basis of the relations (8) and (24), one can obtain, after some transformations, that

$$H_{\tilde{m}_x}^{(q)} = \frac{-i\gamma_m}{|\gamma_m|} H_{m_x}^{(q)},$$
$$H_{\tilde{m}_y}^{(q)} = \frac{i|\gamma_m|}{\gamma_m} H_{m_y}^{(q)}. \quad (30)$$

So the magnetic field has the elliptical polarization analogous to the polarization of the electrical field.

We can consider the relations (24) and (30) as a special type of reflection symmetry for the complex modes that realize active power flow through a chirowaveguide. Such a type of reflection symmetry has to be provided by a special discontinuity at the end of a complex-mode waveguide and is different from reflection symmetry in a plane perpendicular to the Z-axis in an infinite waveguide. The last type of reflection symmetry is described in [19] for propagating or evanescent modes in a two-dimensional (the fields are independent of the X-axis) chirowaveguide. It is not so difficult to show that Maxwell equations (2) for propagating or evanescent guide mode $-m$ will be coincided with Maxwell equations for guide mode m if $\gamma_{-m} = -\gamma_m$ and

$$E_{x_{-m}} = E_{x_m},$$
$$H_{x_{-m}} = H_{x_m}$$
$$E_{y_{-m}} = -E_{y_m},$$
$$H_{y_{-m}} = -H_{y_m}$$
$$E_{z_{-m}} = E_{z_m},$$
$$H_{z_{-m}} = H_{z_m}. \quad (31)$$

Specifically, for evanescent mode ($\gamma_m = \alpha_m, \alpha_{-m} = \alpha_{\tilde{m}} = -\alpha_m$) we, evidently, can see the distinction between field relations defined by (24) and (30) and by (31).

The orthogonality relations (10) and (24) enable us to correlate the parameter of field polarization ellipses with mode propagation constants. Since (23) was obtained for a waveguide without a nonchiral layer ($b = 0$ on Fig. 3), our analysis will be correct for such a kind of a structure.

Let $\gamma_p = \gamma_{\tilde{m}} = -\gamma_m^*$ (see Fig. 2). If $\gamma_m\gamma_p^* \neq k_d^2$, we have from (23)

$$\int_o^a (E_{m_x}E_{\tilde{m}_y}^* + E_{m_y}E_{\tilde{m}_x}^*)\, dy = 0. \quad (32)$$

Using (24) and (28), we obtain for complex mode ($\gamma_m = \alpha_m + i\beta_m$)

$$\left[(a + ib)\frac{|\gamma_m|}{\alpha_m - i\beta_m} - (a - ib)\frac{\alpha_m - i\beta_m}{|\gamma_m|}\right] \cdot \int_o^a |E_{m_y}|^2\, dy = 0. \quad (33)$$

This relation gives the result

$$\frac{a}{b} = -\frac{\alpha_m}{\beta_m}. \quad (34)$$

So we have a correlation between the parameters of the polarization ellipse of complex mode m and the parameters of the propagation constant.

Now we consider propagating and evanescent modes. Let for propagating modes, $\beta_m = \beta_p$. In such a case, we satisfy the relation (32) only if the condition (29) of elliptical polarization takes place. It corresponds to the results in [1]. Let for evanescent modes, $\alpha_m = \alpha_p$. On the basis of (23), we have linear polarization for evanescent modes. Let us have two propagating modes with the propagation constants β_m and β_n ($\beta_m \neq \beta_n$) and the polarizations described as

$$\frac{E_{m_x}}{E_{m_y}} = iA_m;$$
$$\frac{E_{n_x}}{E_{n_y}} = iA_n \quad (35)$$

where A_m and A_n are real quantities. After some transformations, we have from (12) and (23) the following two equations

$$\left.\begin{aligned} \beta_m\beta_n A_m A_n + k_d^2 = 0 \\ \beta_m^2 - \beta_n^2 + 2\omega\mu\nu(\beta_n\ A_n - \beta_m A_m) = 0 \end{aligned}\right\}. \tag{36}$$

These relations show the correlations between the polarization parameters and the propagation constants of two propagating modes.

IV. Conclusion

In this paper we have obtained two types of mode orthogonality in chirowaveguides. The first one is based on the vector equation of the eigenfunction problem. The vector formulated orthogonality (so-called power orthogonality) is a general form of orthogonality.

The second type of mode orthogonality is derived on the basis of scalar equations. We have obtained the scalar formulated orthogonality relations for two parallel-plate isotropic chirowaveguides. It is evident that this type of orthogonality differs from the vector formulated orthogonality for the same structure of chirowaveguide.

In this paper we have given special attention to a problem of complex modes in chirowaveguides. Active power flow in a waveguide may take place by combination of two complex modes of the spectrum. We have obtained the correlation between field components of two complex modes that transfer active power flow. The polarization of complex modes in chirowaveguides is different from the polarization of propagating and evanescent modes. According to our analysis, one can correlate the polarization parameters of modes with their propagation constants.

Acknowledgment

The author would like to thank Prof. A. Hardy for his attention to this paper and the stimulating debates they had in connection to it.

References

[1] P. Pelet and N. Engheta, "The theory of chirowaveguides," *IEEE Trans. Antennas Propagat.*, vol. 38, no. 1, pp. 90–98, 1990.

[2] S. F. Mahmoud, "On mode bifurcation in chirowaveguides with perfect electric walls," *J. Electromagn. Waves Appl.*, vol. 6, no. 10, pp. 1381–1392, 1992.

[3] H. Cory and S. Waxman, "Wave propagation along a fully or a partially loaded parallel plate chirowaveguide," *IEE Proc.*, pt. H, vol. 141, no. 4, pp. 299–306, 1994.

[4] H. Cory and I. Rosenhouse, "Electromagnetic wave propagation along a chiral slab," *IEE Proc.*, pt. H, vol. 138, no. 1, pp. 51–54, 1991.

[5] M. I. Oksanen, P. K. Koivisto, and I. V. Lindell, "Dispersion curves and fields for a chiral slab waveguide," *IEE Proc.*, pt. H, vol. 138, no. 4, pp. 327–334, 1991.

[6] C. Eftimiu and L. W. Pearson, "Guided electromagnetic waves in chiral media," *Radio Sci.*, vol. 24, no. 3, pp. 351–359, 1989.

[7] G. Busse and A. F. Jacob, "Complex modes in circular chirowaveguides," *Electron. Lett.*, vol. 29, no. 8, pp. 711–713, 1993.

[8] P. Pelet and N. Engheta, "Modal analysis for rectangular chirowaveguides with metallic walls using the finite-difference method," *J. Electromagn. Waves Appl.*, vol. 6, no. 9, pp. 1277–1285, 1992.

[9] H. Cory, "Wave propagation along a closed rectangular chirowaveguide," *Microwave Opt. Technol. Lett.*, vol. 6, no. 14, pp. 797–800, 1993.

[10] N. Engheta and P. Pelet, "Surface waves in chiral layers," *Opt. Lett.*, vol. 16, no. 10, pp. 723–725, 1991.

[11] N. Engheta, D. L. Jaggard, and M. W. Kowarz, "Electromagnetic waves in Faraday chiral media," *IEEE Trans. Antennas Propagat.*, vol. 40, no. 4, pp. 367–374, 1992.

[12] Z. Shen, "The theory of chiroferrite waveguides," *Microwave Opt. Technol. Lett.*, vol. 6, no. 7, pp. 397–401, 1993.

[13] N. Engheta and P. Pelet, "Mode orthogonality in chirowaveguides," *IEEE Trans. Microwave Theory Tech.*, vol. 38, no. 11, pp. 1631–1634, 1990.

[14] L. B. Felsen and N. Marcuvitz, *Radiation and Scattering of Waves.* Englewood Cliffs, NJ: Prentice-Hall, 1973, ch. 1.

[15] E. O. Kamenetskii, "Spectral method for modes excitation problem in anisotropic waveguides," *IEEE Trans. Microwave Theory Tech.*, vol. 42, no. 9, pp. 1685–1689, 1994.

[16] A. S. Omar and K. F. Schünemann, "Complex and backward-wave modes in inhomogeneously and anisotropically filled waveguides," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-35, no. 3, pp. 268–275, 1987.

[17] G. L. Veselov and S. V. Raevskiy, *The Layered Metallic Dielectric Waveguides,* Moscow: Radio i svyas, 1988 (in Russian).

[18] R. Marques, F. L. Mesa, and M. Horno, "Nonreciprocal and reciprocal complex and backward waves in parallel plate waveguides loaded with a ferrite slab arbitrarily magnetized," *IEEE Trans. Microwave Theory and Tech.*, vol. 41, no. 8, pp. 1409–1418, 1993.

[19] F. Mariotte and N. Engheta, "Reflection and transmission of guided electromagnetic waves at an air-chiral interface and at a chiral slab in a parallel-plate waveguide," *IEEE Trans. Microwave Theory Tech.*, vol. 41, no. 11, pp. 1895–1906, 1993.

Development of Semi-Empirical Design Equations for Symmetrical Three-Line Microstrip Couplers

Lukang Yu and Banmali Rawat

***Abstract*—Semi-empirical design equations for symmetrical three-line microstrip couplers (TMC's) have been developed. The approach is based on dividing the total capacitance of the system into various basic capacitances, which are then calculated empirically and semi-numerically. The numerical results based on these design equations have been found in good agreement with the previously obtained results.**

I. Introduction

Symmetrical TMC's have been investigated by many authors [1]–[6]. The quasi-static characteristics of these couplers can be completely determined from the capacitance matrix of the structure. For design purposes, a table or a graph for many sets of line parameters has to be prepared. Various methods for calculating the static capacitances inevitably involve time-consuming numerical procedures. This paper intends to derive closed-form expressions for the characteristics of symmetrical TMC's. The approach is based on the division of the total capacitance of the structure into various basic capacitances. These basic capacitances can then be calculated empirically and semi-numerically.

II. Division of the Total Capacitance

The division of the total capacitance of a symmetrical TMC into parallel-plate, fringe, and gap capacitances is shown in Fig. 1. The three propagation modes of the coupler are designated as A-, B-, and C-modes, which correspond to ee-, oo-, and oe-modes, respectively, in [6]. Due to the symmetrical configuration, the vertical centerline of the cross section is replaced by a magnetic wall for A- and B-modes, or an electric wall for C-mode. Using Fig. 1(a), the effective dielectric

Manuscript received January 9, 1995; revised November 27, 1995.
The authors are with the Department of Electrical Engineering, University of Nevada, Reno, NV 89557 USA.
Publisher Item Identifier S 0018-9480(96)01560-8.

0018-9480/96$05.00 © 1996 IEEE